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A SIMPLIFIED ANALYTICAL MODEL OF STIRLING ENGINE EFFICIENCY CALCULATION. A THEORETICAL ANALYSIS OF WORKING GAS

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Abstract:

The indicated power and thermal efficiency are the most important features of the Stirling engine. In the most general way, the efficiency depends on a set of parameters, such as the ratio between the volumes and temperatures of the working gas in the hot and cold compartment, the adiabatic function and energy losses due to friction processes between engine components. Stirling engines can use mono, bi or polyatomic molecules such as helium, hydrogen or carbon dioxide as working gases. The first part of the contribution is dedicated verification and completion of our results obtained on the basis of the technical design data of the Genoa03 Stirling engine. These were determined based on an analytical model of the thermal efficiency of a simplified thermodynamic cycle described in the pressure-volume diagram by two adiabatic transformations and two isochoric transformations. The input data used are those used by our αSETS calculation code. The second part of the article is dedicated to the analytical elaboration of the adiabatic function of the working gas for the case of a mixture consisting of two or three types of working gases at different molar ratios. The adiabatic functions are used to estimate thermal efficiency. It has been shown that a mixture of two or three types of gas does not improve the thermal efficiency of the Stirling engine compared to the use of one-component gas. The results of these models related to the Beale relationship can be used successfully in the design process of Stirling engines.

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1. INTRODUCTION

The Stirling engine is a mechanical assembly consisting of seven distinct compartments, as follows: swept and clearance in compression and expansion, regenerator,

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cooler and heater, respectively [Allan J. Organ, 2010; G. Giannakakis et al., 2017]. These mechanical devices operate on a regenerative thermodynamic cycle after a thermal interval defined by the cooler gas temperature, T_c and the gas temperature of the heater T_h, respectively. However, the specific part of the Stirling engine is the regenerator with the role of borrowing energy from the working fluid during one part of the cycle and reimburses it without losses (ideal regenerator) during another part. Although the actual behavior of the engine differs greatly from this ideal cycle, this is still useful to describe the main principles that guide the operation of the engine.

2. ANALYTICAL MODEL OF THERMAL EFFICIENCY

The energetic and efficiency of Stirling engines characterization of the ideal Stirling engine cycle is based on the first and second principle of thermodynamics that establishes the connection between the mechanical work performed, the amount of heat and the variation of the internal energy of the working gas [Allan J. Organ, 2010; G. Giannakakis et al., 2017; G. Walker, 1973].

The most important factors that describe the analytical model of the thermodynamic efficiency of Stirling engines η_S are the ratios between the volumes (ε) and temperatures (τ) of the working gas in the hot and cold compartment and the constant adiabatic function (γ), respectively.

$$\eta_S = \frac{(\tau-1)ln\varepsilon}{\tau ln\varepsilon+\frac{\tau-1}{\gamma-1}} \tag{1.1}$$

The thermal efficiency determined by this model assumes an a priori defined cycle. However, there are Stirling engine operating models for which the operating cycle is not predefined. Thus, in the modeling process it is considered that only isothermal [K. Hirata, 2014] or adiabatic transformations [J. A. Araoz et al., 2015 a); J. Wrona, M. Prymon, 2016] take place in the expansion and compression compartments of the working gas. These models use the general theory of ideal gases to determine the main specific quantities as well as the pressure-volume dependence. It is worth mentioning that the cycle of isothermal or adiabatic models have a quasi-elliptical shape and are closed in the ideal cycle of the Stirling engine [Allan J. Organ, 2010; G. Giannakakis et al., 2017].

3. ADIABATIC FUNCTION OF WORKING GAS MIXTURES

In the literature, results are reported in which the working gas used in Stirling engines consists of a gaseous mixture where usually the effect is discussed from a narrow point of view [Sajjad A. Salih et al., 2023]. The aim of the second part is to highlight the influence of the mixture of ideal mono, bi and polyatomic working gases in the Stirling engine efficiency. The monoatomic gas molecule has three degrees of freedom for the translation operation. The gas diatomic molecule has three translations and two rotations, so five degrees of freedom, while the polyatomic gas molecule has three degrees of freedom for rotation and three degrees for translation operations, so six degrees of freedom in total. As a result, these are associated with the molecules movement in three-dimensional space. For generalization it is considered a mass gas m consisting of a mixture of N types of gases and each of them has mass m_k. In this context g_k represents the mass ratio and $C_{V,k}$ and $C_{P,k}$ are the molar heat capacity at constant volume and pressure of each component. As a result, the specific heat capacity at constant volume and pressure of the gas mixture are provided by the following equations:

$$C_V = \sum_{k=1}^{N} g_k C_{V,k},\ C_P = \sum_{k=1}^{N} g_k C_{P,k} \tag{2.1}$$

On the other hand, for each type of molecule the molar heats capacity at constant volume and pressure are expressed as a function of the universal constant of the gases:

$$C_V = \frac{i}{2}R,\ \ C_P = \frac{i+2}{2}R,\ \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{i}, \tag{2.2}$$

where each type of molecule has degrees of freedom (i) equal to 3,5 and 6, for mono-, di- and poly- atomic molecules, respectively. Thus, we can directly express the adiabatic gas function. Consider the mass ratios of the three types of gases described by the variables x (mono-), y (bi-) and z (poly-) and using the equations (2.1) and (2.2), the adiabatic function for a gas mixture of mono, bi and tree atomic molecules is given by:

$$\gamma(x, y, z) = \frac{5x+7y+8z}{3x+5y+6z} \tag{2.3}$$

and there is a relationship between the variables of the γ function:

$$x, y, z > 0,\ \ x + y + z = 1 \tag{2.4}$$

In the mathematical sense [G. Strang, E. J. Herman, 2020], the equation (2.3) is a function of three variables that cannot be manipulated graphically in Cartesian coordinates. In this context, the adiabatic function for a two-component mixture of type (mono-, bi-), (mono-, tri-) and (di-, tri-) can be analyzed. Simplified forms of the equation (2.3) given by the functions $\gamma_{xy}=\gamma(x,y,0)$, $\gamma_{xz}=\gamma(x,0,z)$ and $\gamma_{yz}=\gamma(0,y,z)$ are used in the evaluation of the adiabatic function for two-component mixtures. The coordinates that cancel the first partial derivatives of the adiabatic functions γ_{xy}, γ_{xz} and γ_{yz} could provide the values of the ratios of the two mixture components for the optimal thermal efficiency.

4. RESULTS AND DISCUSSIONS

Our results are based on input data describing the technical characteristics of the Genoa Stirling engine. It is a prototype for research studies by GENOA 03 Stirling Engine S.R.L. Company with an electric power of 2.9 kW [Genoastirling S.RL., 2014]. According to its specifications, it can supply electricity using various mono bi or polyatomic gases as working gas. It can develop a rotational speed of up to 600 rpm and the cold sink and hot source temperatures around 293 K and 1023 K. respectively. The working gas pressure was considered 3 MPa. From a thermodynamic point of view, hydrogen is the best, however, it is not safe to work, while helium and carbon dioxide are safer to use. The results of this study are summarized in the Table 1. First, they are based on the technological data of the Stirling engine [M. T. García et al., 2018; J. A. Araoz et al., 2015, b)], as well as on the adiabatic constants of pure working gases that are contained in the first two lines of the Table 1. These are input data for our αSETS computer code. Stirling engine performances are analyzed using computational code that performs a complete modeling and simulation based on isothermal and adiabatic theories [A. Bot et al., 2018]. The computer code performs an assessment of the quantitative effects of heat transfer and friction effects due to working gas flows in the cooling, regenerator and heating compartments. As a result, the following four lines of the Table 1 contain the code output data, such as the working gas temperatures in the cooler and heater, temperatures and volumes ratio. These data are required for thermal efficiency calculation. In the following two rows are given the values of thermal efficiencies calculated from αSETS computer code and based on the equation (1.1).

Table 1. Temperatures, volumes of working gases obtained from αSETS code, thermal efficiencies, minimum and maximum values of the adiabatic function for two-component working gas mixtures.

Working gas	**He**	**H_2**	**CO_2**
Adiabatic constant γ	1.66	1.41	1.3
Cooler gas temperature T_c (K)	321	313	337
Heater gas temperature T_h (K)	997	1002	989
Volume ratio (ε)	3	3	3
$\tau=T_h/T_c$	3.09	3.20	2.93
Thermal efficiency (%) [αSETS]	29.7	30.6	25.1
Thermal efficiency (%) equation (1.7)	34.9	27.2	21.9
	mixture He-H	mixture He-CO_2	mixture H-CO_2
min $\gamma_{x,y}$, $\gamma_{x,z}$, $\gamma_{y,z}$	$(x=0,y=1,\gamma_{x,y}=1.41)$ $(x=0,y=0,\gamma_{x,y}=1.41)$	$(x=0,z=1,\gamma_{x,z}=1.3)$ $(x=0,z=0,\gamma_{x,z}=1.3)$	$(y=0,z=1, \gamma_{y,z}=1.3)$ $(y=1,z=0, \gamma_{y,z}=1.3)$
max $\gamma_{x,y}$, $\gamma_{x,z}$, $\gamma_{y,z}$	$(x=1,y=0,\gamma_{x,y}=1.66)$ $(x=0,y=0,\gamma_{x,y}=1.66)$	$(x=0,z=0,\gamma_{x,z}=1.66)$ $(x=1,z=0,\gamma_{x,z}=1.66)$	$(y=0,z=0,\gamma_{y,z}=1.41)$ $(y=1,z=0, \gamma_{y,z}=1.41)$

There are some differences between the thermal efficiency of the Stirling engine determined based on the ideal working gas model developed in section 2 and that obtained based on the αSETS code in which a complete analysis is performed. For the calculation of the real thermal efficiency there are also additional parameters that have been considered in αSETS code [A. Bot et al., 2018]: (i) the dynamic viscosity and thermal conductivity of the working gases; (ii) conductive heat transfer of the main engine parts; (iii) geometry and thermal properties of the regenerator; (iv) pressure losses in the engine compartments due to the viscosity of the working gas. This can explain the different efficiency values obtained with αSETS code and using the simplified model from equation (1.1) and the inversion of the efficiencies between hydrogen and helium considered as working gases.

The first results presented in Figure 1 determine the thermal efficiency calculated using the equation (1.1). The intervals used to define the temperature and volume ratios are centered on the values of the parameters τ and ε determined by the αSETS code. The graphical analysis shows that the thermal efficiency increases significantly for the increase of the values of the adiabatic constants of the working gas and of the ratios τ and ε, respectively. This can be justified by analyzing the gradient of the vector fields and the contours of the η_S function.

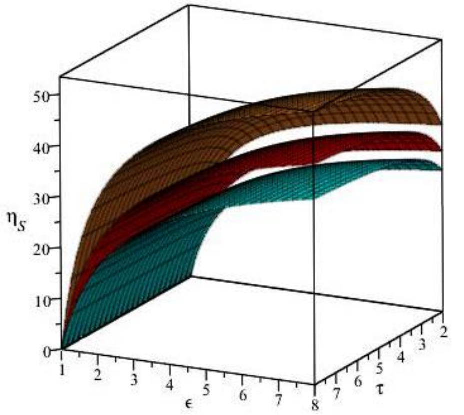

Figure 1. Thermal efficiency of ideal Stirling engine, working gas: brown – helium, red – hydrogen, cyan –carbon dioxide

Figure 2 shows the gradient vectors that are always perpendicular to the contours of the η_S function. The contour lines are used to illustrate points of equal altitude above the level $\eta_S(\tau, \varepsilon) = 0$. The length and the direction of the gradient vectors in the plane, (τ, o, ε) give a complete image of the relative variation for the thermal efficiency. Therefore, the information contained in Figure 2 may have important technical implications in the design of Stirling engines. Before analyzing the practical validity of the thermal efficiency given by the equation (1.1), it is worth mentioning that the example in the paper [K. Hirata, 2014] based on the isothermal model offers a thermal efficiency identical to that of the Carnot cycle which considers only the ratio between the two temperatures τ, without to take into account the volume ratio and the adiabatic function of working gas. Moreover, the same example considers the two expansions and compression volumes equal, which from a technical point of view is not common.

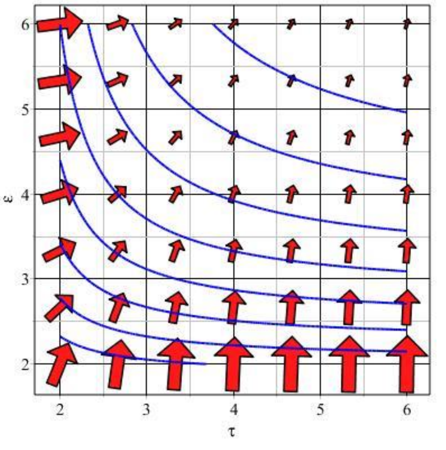

Figure 2. Gradient of the vector fields and the contours of the η_S function

The model of thermal efficiency expressed by equation (1.1) shows if the ratio of the two volumes provided by the parameter ε is equal to one, then the thermal efficiency of such a Stirling engine is zero, consequently from a technological point of view such an engine cannot exist. The designer of a Stirling engine first aims to obtain an indicated power established a priori. The type of working gas, its temperatures, heater swept volume and thermal efficiency can be easily designed, but it is necessary to adjust the cooler swept volume. This can be easily estimated from the equation (1.1) resulting from the developed model. A final step in the Stirling engine design process is to check the indicated power initially proposed. This can be done from the well-known Beale relationship. It is based on an empirical correlation to predict the output power of a motor expressed by the below equation,

$$W_0 = B_n P V_C f \tag{3.1}$$

where W_0, B_n, P, V_C and f are indicated power, Beale number, average gas pressure, swept volume of the power piston and engine cycle frequency, respectively. The original Beale number is usually assumed to be in the range of 0.01-0.015. Let us a case study when designing a Stirling engine with construction data summarized in the third column of the Table 1, but for which we assume we do not know the value of cooler swept volume. Then, for a desired value of the output power determined by Beale's relation, the value of the cooler swept volume can be estimated. From a simple calculation for an output power of about 3 kW, the colder swept volume should be projected about three times smaller than the heater swept volume, which is a good match for the experimental Genoa Stirling 03 engine model [Genoastirling S.RL., 2014]. This observation shows that even the simplified model presented in this contribution can be used successfully in the design process of Stirling engines.

The second set of results refers to the estimation of adiabatic function but for mixtures of mono, bi and polyatomic gases in various mass ratios. The results show that the process of using a gas mixture in the operation of the Stirling engine does not lead to an increase in thermal efficiency. The results show the role played by the adiabatic function, described by the equations (2.3) and (2.4), corresponding to two-component working gas mixtures, in the thermal efficiency of Stirling engine. The behavior of the adiabatic functions for bicomponent gas mixtures are shown in Figure 3. On can see that the described surfaces do not have local vertex points. The lack of local vertex points further demonstrates that there are no favorable concentrations of two-component working gas mixtures leading to special thermal efficiencies. The minimum and maximum values of the adiabatic function for the considered two-component working gas mixtures are located on the lines of the graphical representations contained in Figure 3. These values are defined by the coordinate pairs shown in the last lines of the Table 1. The horizontal surfaces in Figure 3 falling into hatched and colored rectangular triangles determine the concentrations of the two-component mixtures (1,2), (1,3) and (2, 3) of the working gases which are in fact the defining fields of adiabatic functions. Furthermore, the adiabatic constants of pure and mixed working gases is in the ordering relation: $\gamma_y \leq \gamma_{x,y} \leq \gamma_x$, $\gamma_z \leq \gamma_{z,x} \leq \gamma_x$ and $\gamma_z \leq \gamma_{z,y} \leq \gamma_y$. These relations imply that the thermal efficiency for the same ratios of temperatures and volumes is in the same ordering relation.

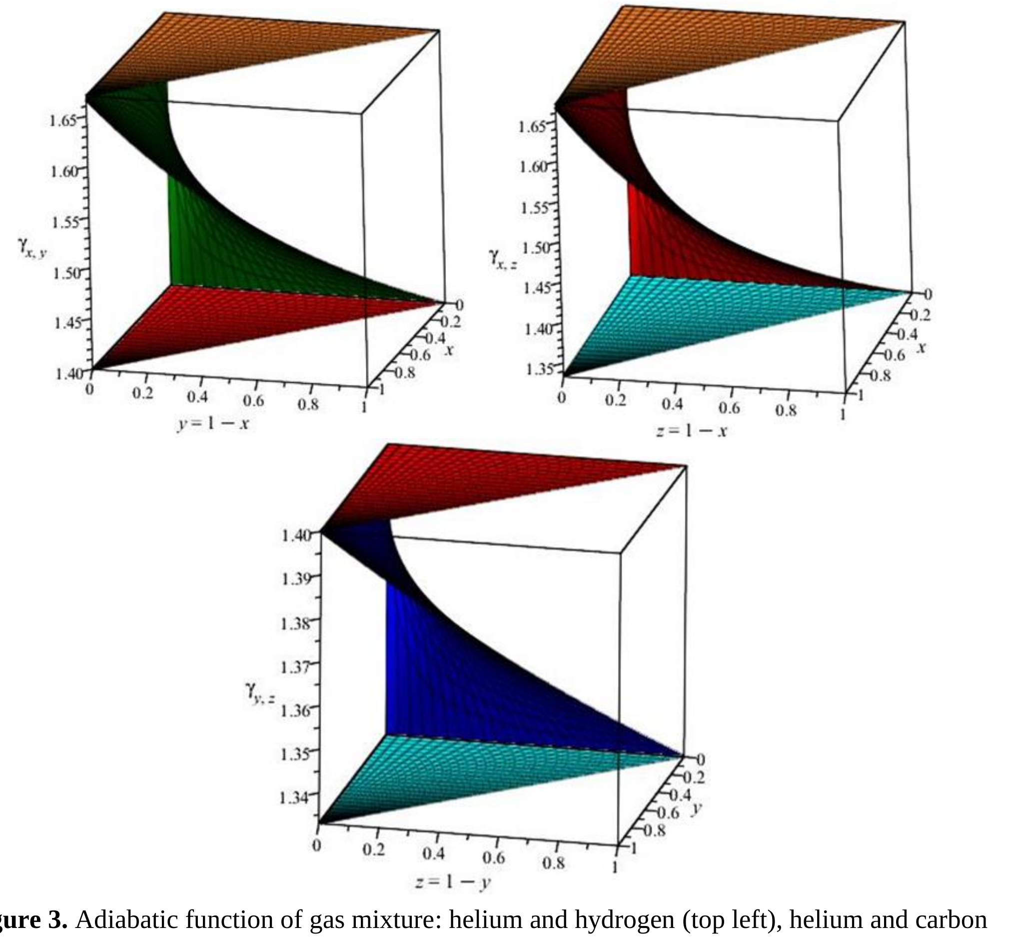

Figure 3. Adiabatic function of gas mixture: helium and hydrogen (top left), helium and carbon dioxide (top right) and hydrogen and carbon dioxide (bottom)

The estimation of adiabatic function for mixtures of three gases, mono (He), bi (H_2) and polyatomic (CO_2), in various mass ratios, are presented in Figure 4.

The maximum of adiabatic function corresponds to the ratio values of y=0, z=0, x=1 (pure helium gas). If the adiabatic constants of the three gases are γ_x, γ_y and γ_z (with $\gamma_z < \gamma_y < \gamma_x$) than the adiabatic constant of the mixture $\gamma_{x,y,z}$, regardless the ratios, will be $\gamma_z < \gamma_{x,y,z} < \gamma_x$.

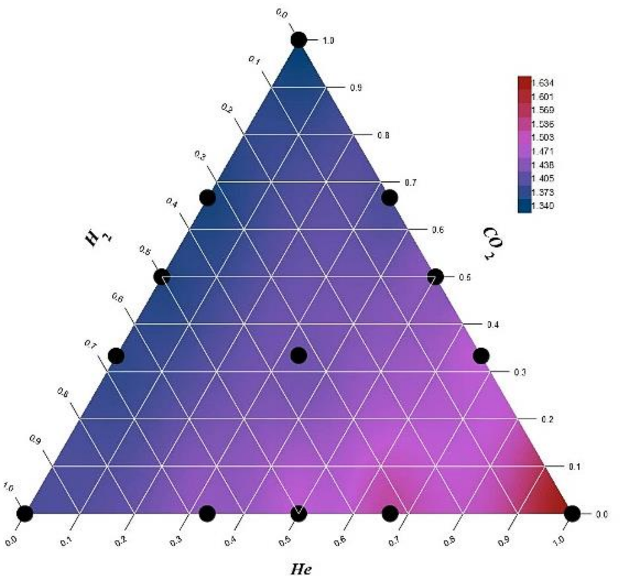

Figure 4. Adiabatic function of three gas mixture helium, hydrogen and carbon dioxide

5. CONCLUSIONS

We showed that the use of two-or three- components gas mixture is not recommended. Pure gases with high adiabatic constants are more suitable to be used as working gas in the Stirling engines. Nevertheless, there are also other factors that are influencing the Stirling engine efficiency (the dynamic viscosity and thermal conductivity of the thermal agent, pressure losses in the engine compartments due to the viscosity of the thermal agent, etc.) that will be discussed in a future contribution.

Author contributions: N. Aldea used the simplified analytical model for the Stirling engine efficiency analysis. F. Matei designed the mathematical model of adiabatic function. V. Rednic contributed to the αSETS code implementation.

Conflict of Interest Statement: The authors declare no conflicts of interest associated with this manuscript.

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